Scalable solution methods via optimal control reformulation

Denis Ridzal[*] and Pavel Bochev[†]

We consider an optimization–based approach for the scalable solution of PDE problems comprised of multiple physics operators with fundamentally different mathematical properties. The approach relies on ideas from optimization and control to transform the solution of the composite multiphysics problem into the solution of a sequence of problems governed by scalable components.

The proposed optimization–based framework relies on three essential steps. First, an appropriate operator decomposition is applied to the original composite problem, breaking it down into component physics problems for which scalable solvers are available. Second, the components are coupled via distributed control parameters, used e.g. in the case of single–domain problems, and/or boundary control parameters, used e.g. in the context of multidomain problems, and a suitable objective functional. Third, the resulting large–scale PDE–constrained optimization problem is solved either directly as a fully coupled algebraic system, or in the null space of the PDE constraints.

To demonstrate the potential of the optimization approach we use a model advection–diffusion equation. Efficient multilevel solution of linear systems arising in the discretization of such PDEs is still largely an open problem. This is especially true for algebraic multigrid techniques, where it is not clear what the best approach is to coarse–grid construction for problems that are nearly hyperbolic. In contrast to classical operator–split techniques which alternate between between pure advection and pure diffusion solves, our approach breaks down the solution of the advection–diffusion equation into the solution of a sequence of diffusion dominated problems, each of which can be efficiently handled by standard multilevel techniques. First, an additive operator split is applied to the original advection–diffusion problem. Second, the resulting equation is reformulated as a PDE–constrained optimization problem, which is solved in the null space of the constraint operator. Preliminary numerical results will be presented.

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[*]Sandia National Laboratories, Optimization and Uncertainty Quantification, P.O. Box 5800, MS 1320, Albuquerque, NM 87185, USA, dridzal@sandia.gov

[†]Sandia National Laboratories, Applied Mathematics and Applications, P.O. Box 5800, MS 1320, Albuquerque, NM 87185, USA, pbboche@sandia.gov